Portfolios of Efficient Frontier Testing by The Second Order Stochastic Dominance

Feng-Jui Hsu[a] and Tsai yi Wang [b,*]
Department of Finance, National Chung Hsing University, Taiwan
E-mail: rickhsutw@yahoo.com.tw
No.250, Guoguang Rd., South Dist., Taichung City 402, Taiwan (R.O.C.), 40227

Abstract

This article uses the second order stochastic dominance (SSD) to choosing the portfolio which is the efficient frontier generating by expected return and variance. The finding indicates that the portfolios of efficient frontier might be ranking by the stochastic dominance, but the results only shown that is opposite the efficient frontier and is difficult to ranking by various weighted allocation portfolios. Moreover, the most significant difference by p-value was in the middle of mean and variance range. With the second order stochastic analysis, the investor or fund manager can select the portfolio by SSD. If investors are difficult to define risk preference for investment strategy, they have the alternative of choosing the portfolio by SSD.

Key Words: Efficient Frontier, Stochastic Dominance, Second Order Stochastic Dominance

Introduction

Portfolios of efficient frontier (or mean-variance method) developed by Markowitz (1952) and extended by Merton (1972) and Vörös (1986). Most of fund managers use the mean-variance (Chamberlain & Rothschild, 1983) optimization to solving an asset allocation which minimizes the portfolio risk (Porter & Gaumnitz, 1972; Joy & Porter, 1974; Koehn & Santomero, 1980) with a certain stock return. If a portfolio is efficient, there is no other portfolio with same variance that has a larger mean and no other portfolio with same mean that has a smaller variance (Tolley & Pope, 1988). The investors or fund managers can pay attention on the efficient subset of portfolios by their risk preference.

Recently, many authors were devoted to the study of portfolio choice. The stochastic dominance (SD) (Whitmore, 1970; Ogryczak, & Ruszczyński, 1999) approach is the one of the important methods to the theory of portfolio choice. Stochastic dominance is most commonly applied in finance and the economics of uncertainty (Kjetsaa & Kieff, 2003). Few of stochastic dominance applications to portfolio selection have take advantage to develop or construct a portfolio of risky assets.

The objective of this paper is to use the second order stochastic dominance (SSD) (Prakasa, 1994; Leitner, 2005; De Giorgi & Post, 2008) to choosing the portfolio which is the efficient frontier generating by expected return and variance. However, the portfolios of efficient frontier differ significantly from each other by SSD method; if portfolios of efficient frontier can be ranking, the SSD can be an alternative of choosing the portfolio without risk preference.

The Stochastic Dominance Model

The objective of portfolio management is to using the second order stochastic dominance to choose the portfolio from the efficient frontier. The basis of mean-variance method is considered as the same

efficient portfolio in the frontier. The fund managers usually advise investors to choose the frontier portfolios by investor's risk preference. The paper attempts to examine the stochastic dominance for each of the portfolios on the efficient frontier, and portfolio choice can be determined basing on the stochastic dominance not depending on the risk preference. The principle of the second order stochastic dominance (SSD) is state as follows:

$$D(t) = \int_{-\infty}^{t} F_A(x)dx - \int_{-\infty}^{t} F_B(x)dx \qquad (1)$$

where A and B represent the populations. The hypotheses to be tested are

H_1: B is not second-order stochastically dominant over A.

H_2: B is second-order stochastically dominant over A.

Each of the selected portfolios by mean-variance frontier will be ranking under the second order stochastic dominance. For stochastic dominance efficiency, the out-sample data will be evaluated from stock market data in next year.

Stochastic Dominance in Efficient Frontier

Market Data

Daily data of stock price from 30 the largest market value stocks for the period 2004-2005 in U.S. market were used for in-sample and out-sample data, and the 30 companies are selected based on the ranking of company's market value. The first year database was used to generate a mean-variance efficient frontier of portfolios and to ranking the portfolios by the second order stochastic dominance, and the second year database was used as out-sample for evaluating the ranking. All daily data come from the Thomson Financial DataStream database.

Efficient Frontier Portfolios

The optimal portfolio should be lying on the efficient frontier. The stocks returns and covariance should be specified for mean-variance efficient frontier. Given a risky assets of 30 stocks, and computes a portfolio of asset investment weights that minimize the risk for given values of the expected return. 10 portfolios were selected along the efficient frontier basing on the 30 risky assets. These results are shown in Figure 1.

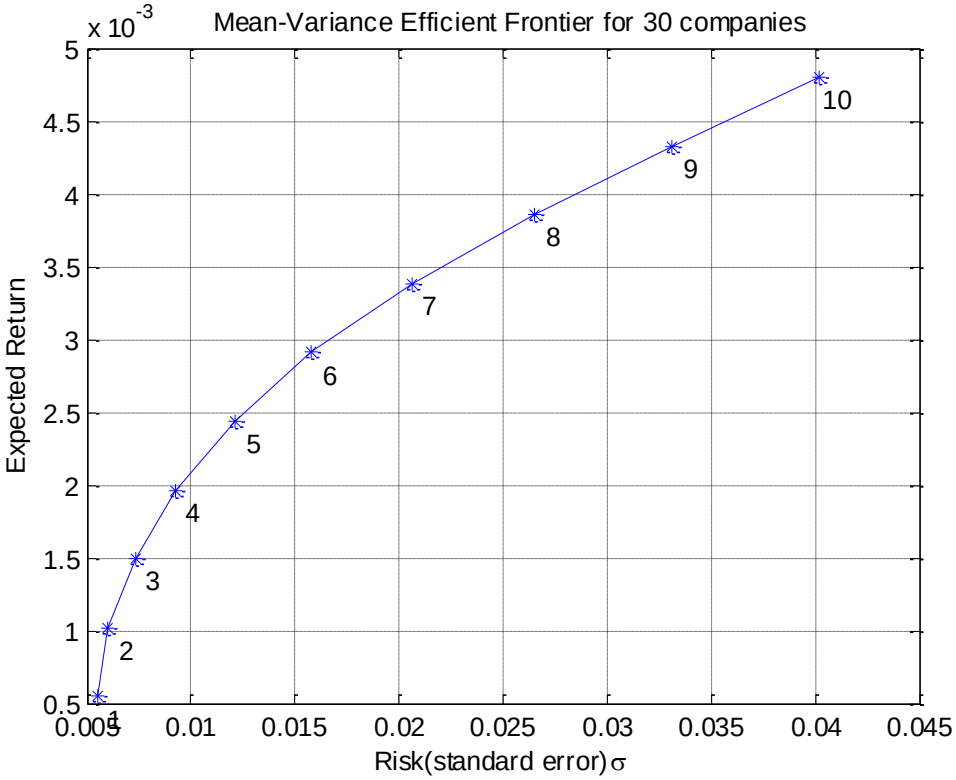

Figure 1. Efficient Frontier for 30 companies combination

Statistical Test for Stochastic Dominance

Based on the 10 portfolios lying on the efficient frontier, a comparison of two portfolios will be testing for the second order stochastic dominance. The data generating process is based on the Bootstrapping to simulating p-value. The results of p-value for portfolios comparison are shown in Table1. If the critical p-value is set to be 0.01, the portfolio 4 dominates portfolio 5, 6, 7, 8, 9, and 10. The portfolio 6 dominates portfolio 7, 8, 9, and 10, and the p-value are significantly larger than 0.01. The table 1 has shown that the portfolios of efficient frontier are definitely inefficient depending on p-value. The following subsection will test the out-sample data by the same weighting of 10 portfolios, whether the mean-variance analysis is consistent or not with the second order stochastic dominance.

Table 1. Summary of p-value for stochastic dominance of the portfolios

	Portfolio								
Portfolio	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
2	0.001	0	0	0	0	0	0	0	0
3	0.001	0.003	0	0	0	0	0	0	0
4	0.003	0.004	0.008	0	0	0	0	0	0
5	0.001	0.006	0.006	0.032	0	0	0	0	0
6	0.004	0.011	0.016	0.026	0.041	0	0	0	0
7	0.008	0.025	0.028	0.054	0.072	0.101	0	0	0
8	0.019	0.022	0.028	0.052	0.073	0.097	0.118	0	0
9	0.025	0.04	0.041	0.056	0.074	0.112	0.119	0.128	0
10	0.031	0.049	0.052	0.068	0.076	0.105	0.127	0.132	0.138

Note: a number in the table means that the p-value of the listed in top row portfolios is second order stochastic dominance the listed in left column portfolios by 1000 repetition. For example, the portfolio 6 dominates the portfolio 7 that p-value is 0.101. The data generating process is based on the Bootstrapping to simulating p-value.

Out-Sample Test for Mean-Variance Analysis

10 portfolios lying on the efficient frontier might have the same efficiency for out-sample test. Beginning with the 30 stocks for 2005, the mean and variance can be evaluated by the different weighting for 10 portfolios. If a portfolio is still efficient by using out-sample data, no other portfolio with same mean has a smaller variance or no other portfolio with same variance has a larger mean. The out-sample test results are shown in Figure 2. The portfolios 1-6 seem to retain frontier but those are not sure of the efficiency, and the portfolios 7-10 are opposite to the efficient frontier. Table 1 has shown that the best performance of the return is the portfolio 6 which significantly dominates the portfolios 7, 8, 9, and 10. The portfolio 5 also dominates the portfolios 6, 7, 8, 9, and 10 significantly. The portfolio 5 and 6 seem to be the best choice by SSD.

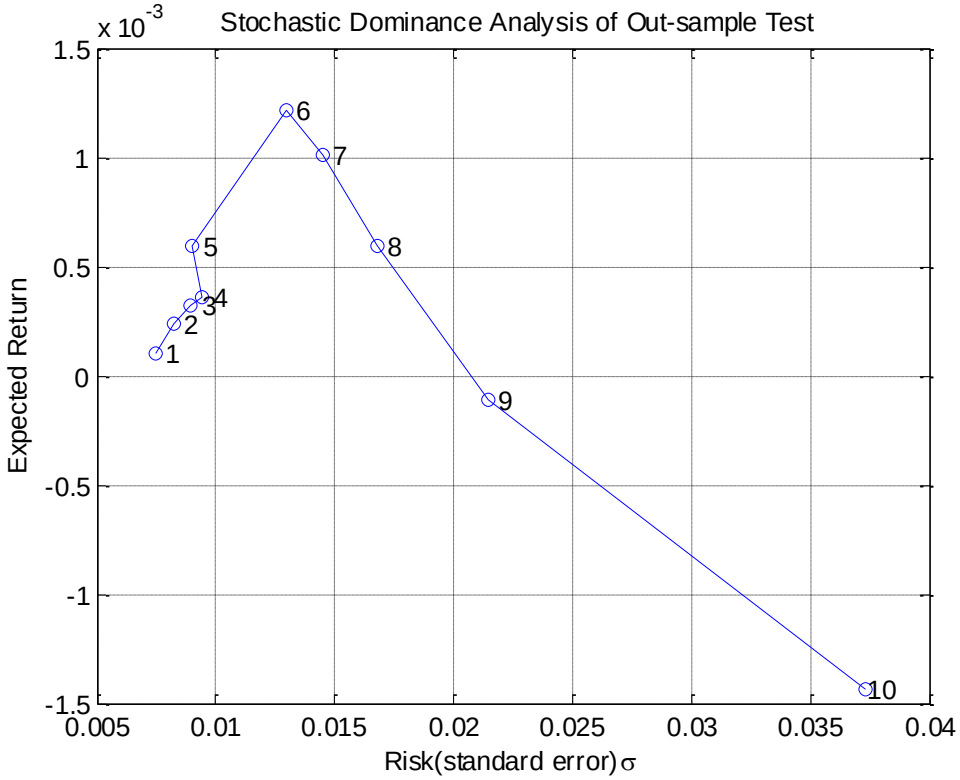

Figure 2. Mean-variance test for out-sample data

Conclusions

The second order stochastic dominance was used to test the portfolios of the mean-variance efficient frontier. The portfolios of efficient frontier should be inconsistent by SSD analysis that has been expected. The portfolios of efficient frontier might be ranking by the stochastic dominance, but the results only shown that is opposite the efficient frontier and is difficult to ranking by various weighted allocation portfolios. The most significant difference by p-value was in the middle of mean and variance range. For future study, the efficient frontier of portfolios should be use the whole market data for optimalization, and the critical p-value calculation based on the Bootstrapping should be clearly defined for ranking purpose. With the second order stochastic analysis, the investor or fund manager can select the portfolio by SSD. If investors are difficult to define risk preference for investment strategy, they have the alternative of choosing the portfolio by SSD. The fund manager may use SSD to construct the portfolio with efficient frontier. The finical strategy can be a active management following the SSD analysis.

References

Chamberlain, G., and M. Rothschild. "Arbitrage, factor structure, and mean-variance analysis on large asset markets." *Econometrica*, Vol. 51, No. 5 (1983), pp. 1281-1304.

De Giorgi, E., and T. Post. "Second-order stochastic dominance, reward-risk portfolio selection, and the CAPM." *Journal of Financial and Quantitative Analysis*, Vol. 43, No. 2 (2008), pp. 525-546.

Joy, O. M., and R. B. Porter. "Stochastic Dominance and Mutual Fund Performance." *Journal of Financial and Quantitative Analysis*, 9 (1974), pp. 25-31.

Leitner, J. "A Short Note on Second‐Order Stochastic Dominance Preserving Coherent Risk Measures." *Mathematical Finance*, Vol. 15, No. 4 (2005), pp. 649-651.

Kjetsaa, R., and M. Kieff. "Stochastic dominance analysis of equity mutual fund performance." *American Business Review*, Vol. 21, No. 2 (2003), pp. 1-8.

Koehn, M., and, A. M. Santomero. "Regulation of bank capital and portfolio risk." *The Journal of Finance*, Vol. 35, No. 5 (1980), pp. 1235-1244.

Markowitz, H. "Portfolio selection." *The Journal of Finance*, Vol. 7, No. 1 (1952), pp. 77-91.

Merton, R. C. "An analytic derivation of the efficient portfolio frontier." *Journal of Financial and Quantitative Analysis*, Vol. 7, No. 4 (1972), pp. 1851-1872.

Ogryczak, W., and A. Ruszczyński. "From stochastic dominance to mean-risk models: Semideviations as risk measures." *European Journal of Operational Research*, Vol. 116, No. 1 (1999), pp. 33-50.

Porter, R. B., and J. E. Gaumnitz. "Stochastic dominance vs. mean-variance analysis: An empirical evaluation." A*merican an Economic Review*, 62 (1972), pp. 438-446.

Prakasa, B. "Testing for second-order stochastic dominance of two distributions." *Econometric Theory*, 10 (1994), pp. 849-866.

Tolley, H. D., and R. D. Pope. "Testing for stochastic dominance," *American Journal of Agriculture Economics*, 70 (1988), pp. 693-700.

Vörös, J. "Portfolio analysis—an analytic derivation of the efficient portfolio frontier." *European journal of operational research*, Vol. 23, No. 3 (1986), pp. 294-300.

Whitmore, G. A. "Third-degree stochastic dominance." *The American Economic Review*, Vol. 60, No. 3 (1970), pp. 457-459.

Authors Bio

Feng-Jui Hsu[a] and Tsai-yi Wang [b,*]

[a]Department of Finance, National Chung Hsing University, Taiwan, Assistant Professor
[b,*] Department of Insurance and Finance, National Taichung University of Science and Technology, Taiwan, Assistant Professor

Address:
[a]No.250, Guoguang Rd., South Dist., Taichung City 402, Taiwan (R.O.C.), 40227
[b,*]No.129, Sec. 3, Sanmin Rd., North Dist., Taichung City 40401, Taiwan (R.O.C.)

Email:
[a]rickhsutw@yahoo.com.tw
[b,*]peterw@nutc.edu.tw

Telephone Number:
[a] +886-4-22840808 Ext.446
[b,*] +886-4-22915922

Short bio:
[a] Department of Finance, National Chung Hsing University, Taiwan, Assistant Professor. The research field is corporate governance, portfolio management, and international investment.
[b,*] Department of Insurance and Finance, National Taichung University of Science and Technology, Taiwan, Assistant Professor. The research field is earning management and insurance.