Plato’s Mathematical Forms: An Islamic Critique

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Abstract

Plato’s Theory of Forms is grounded on the belief that these forms are representatives of eternal concepts that belong to a transcendent realm separated from the physical or the material realm. The Theory of Forms which can also be understood in terms of mathematics, held mathematical knowledge in high esteem as these forms collectively merged, depict the Highest Form of Goodness or God. However, Plato’s mathematical forms, since the time of Aristotle, and to date, had been refuted and amongst the point of attack, is the inconclusive justification by Plato in addressing and linking the concept of God and the true existence of these forms. Hence, this paper seeks to address this issue by providing an Islamic critique in an overview of Plato’s mathematical forms.

Keywords: Platonic forms, Mathematical, Theory of Forms

1. Introduction

Plato’s Theory of Forms or Perfect Ideas which forms the central theme of his philosophy are being defined as having abstract properties and qualities. According to Plato, the forms are by nature, transcendent and do not exist in space and time. For example, there is no particular place or time at which roundness or redness exists as the forms of roundness and redness can be found in many particular spatial locations. Even if all round and red objects perish, their properties would still exist since there is no particular place or time at which these forms or properties exist.

The forms or ideas being also eternal and unchanging are independent of sense perception. Since they represent the absolutely true definitions of concepts and exemplify the highest level of the intellect, an understanding of them is only through reason and not empirical investigation. And, for one to truly understand the universe, it is necessary to understand these forms.

As opposed to the ideal nature of these forms, Plato however, contends that the physical world is merely an imperfect image of this world of forms as nothing in the visible world is ever perfect in its kind. Hence, the forms or ideas which collectively are governed by the ‘Form of The Good’ or the highest and purest intellect also consist of the following entities: numbers and the objects of geometry such as lines, points and circles which are apprehended only by reason. Plato used the Greek term eidos to refer to these abstract entities, a word that is often translated as idea or form.
To support his notion of the eternal and abstract nature of these forms, Plato claims that the forms are in opposites of the world of sense experience which is subject to constant change. This material world perceived through the senses, is not real as it is constantly undergoing change. Plato also described any particular object of the physical as “participating” or “partaking” in the world of forms and can be known insofar as we can identify them by a certain “form”. As members of a class of things which share the same form, objects having the same qualities can be made knowable to one. Thus, the ‘forms’ constitute knowledge and, having necessarily, the following characteristics: eternal, immutable, and unchanging.

The forms to Plato, are intelligible and contain universal abstract objects, including numbers such as 2+3=5. Not only is Plato’s forms representing mathematical truths, it is also a manifestation of morality, in which the moral form of justice is hence, a manifestation of these form.

2. Platonic Forms and Mathematics

Plato’s theory of Forms can also be understood in terms of mathematical knowledge. This can be seen in his belief that mathematics is the transition between the physical and the metaphysical realms. As such, mathematics to Plato, deals with matters of truth and ultimate reality as the discipline is a priori in nature. The relations of numbers and the development of the number system are logically necessary, and not temporal. They belong to the world of eternity and immutability. But the relations of things and the development of the universe of things are contingent, a posteriori and temporal (Afridi and Khan 2007). The objects of mathematics are depicted in forms such as triangles, circles and other mathematical objects which require rational understanding or the intellect and the process of logical abstraction. Mathematics by nature being a priori and being most certain and real, could not exist in the material world. Hence, Plato believes that they can only exist in the realm of the ‘forms’.

The importance of mathematical knowledge in the Platonic and Greek tradition can be seen in the discipline being held in high esteem. In the Academy set up by Plato, mathematical knowledge was given the highest priority for students’ entry into his institution of learning. Mathematics was believed to be the most certain knowledge as mathematics exemplifies the pure forms of intellect as one does not seek its truth by relying upon sense perception and experience. For instance, one can know truths such as 4+4+8 without having to check one’s experience of the material world, thus, reducing mathematics as having an abstract existence.

Plato’s view of mathematical knowledge as residing in the metaphysical realm is seen in his claim that mathematical forms are governed by God or what he called, “The Highest Form of Good”. The objects of the mathematician's knowledge are forms, such as triangles, circles, and other mathematical objects do not vary with changes of the visible world (they are constant) and they exemplified the pure forms of intellect.

However, Plato could not account for a very clear explanation of how the universal forms of intellect "participates" in the particular objects of the physical world and thus his theory remains obscure. Plato had not come up with a logical explanation of how these forms are connected to objects of the physical world with the exception of merely positing the claim that they have an abstract existence and are mental representations of visible things in the physical realm.

On his attempt to make this point clear, Plato in the Republic, compared the Idea of the Good or God to the sun which he described as the supreme cause of all knowledge and existence. Thus, the ‘Forms’ or ‘Ideas’ representing objects in the physical world are unified under this Idea of the Good. The idea of the Good or the Sun is believed to be the source of energy which sustains life in all living things in the world.
Hence, in the Platonic system, the ‘Idea of the Good’ is the supreme reality on which all other ideas and all ethical, logical and aesthetic values of the sensible world depend. The ‘Idea of the Good’ is the reality through which the world of becoming is made possible and rational.

However, understanding Plato’s concept of God as embraced in the form of the Good are not without difficulties. His doctrine remains obscure to the present time as there is indeed great difficulty in grasping and understanding Plato’s meaning of the ‘forms’, its mathematical underpinnings and its representation of the highest intellectual order constructed upon a metaphysical structure, the latter somewhat remains mystical.

To Plato, Mathematics, holding a prominent place is treated with esteem as it is the manifestation of pure reasoning and insight into the world of forms. Be it as it may, the difficulty and obscurity in grappling with Plato’s Theory of Forms is finding a clear explanation to the question of “What indeed is his real doctrine? There are several points of ambiguity inherent in the ‘Theory of forms’ that can be addressed in the following questions: How does Plato resolve the claim that the ‘forms’ being eternal and unchanging entities, and, only to be apprehended through rational and logical thinking, are also metaphysical in nature? Since the reality of metaphysics are beyond the ordinary experience and thinking of man, how is the belief in God which rests on these eternal forms justified?

For Plato, the world has a soul and mathematics manifests the concept of God. In the light of this claim, the following argument raises doubt on the validity of Plato’s theory.

First and foremost, if the forms can only be understood through pure reasoning, then necessarily, God is purely cognitive in nature as the knowledge of God can only be attained using the intellect. But, God as envisioned by Plato is also the Idea of the Good (Demiurge) and having a metaphysical connotation of the soul while mathematics, is also a manifestation of the concept God. So, how does Plato account for the logical explanation of the metaphysical (beyond ordinary sense experience) and intellectual realms pertaining to the ‘forms’ or Ideas to that of God and mathematics? Or to be specific, how does he explain explicitly the idea of God being a symbolic expression of the idea of the good? There had never been an explicit explanation on the harmoni relation between these entities and as such, the place of God in the Platonic system had been and still remains the subject of long controversy among scholars. This is so as Plato had at no point made explicit on the causal relation of God with the good or the Form of the Good. There is no one clear definition and concept of the Platonic God and mathematical forms as they can be interpreted from various standpoints which can be surmounted as being contemplative, intuitive, mystical and most of all intellectual. Since Plato’s ‘forms’ draw heavily on the intellect, god in the Platonic-Greek belief is reduced to a mere logical abstraction. As such, a complete understanding of the notion of god can only be achieved by one whose mental and intellectual level is of the highest level. In this context, Plato’s God is totally dependent on reasoned belief but not faith. Plato’s God/Demiurge/Form of the Good being the unifier of the multiplicity of the eternal Forms, necessarily, must possess the necessary characteristic, that is, belonging to the realm of unchanging and absolute realm.

As one of the objects existing in this physical world, human individual being must necessarily have a ‘form’ of its own nature as exemplified by other forms. So, if human beings have mental representations of themselves as manifested in the “forms”, and, if these forms are unified under the Form of the Good or God, necessarily, the Form of the Good must belong to the eternal realm. If this is taken to be true by Plato, how does Plato resolve the question of, “What is the difference between forms representing humans (as a class of objects in the universal world) and other visible things and the form of godliness itself? How can the “lesser” forms reside in the same eternal realm of the Form of the Good or God? If, this is
claimed to be true by Plato, God cannot have either knowledge of the human physical existence as their ‘forms’ and the ‘form’ of the divine power are of the same category. In this sense, divinity is definitely not an attribute of God. In fact, Plato repeatedly claimed that since the Form of the Good or God can be fully understood epistemologically, by some great minds and intellect, then surely, these minds are godly minds! As a stark contrast to the Islamic concept and belief in God, Greek pantheistic notion of God is rather confusing as the attributes of God are also likened to that of the human mind and the rules of logic and mathematics.

As had been addressed earlier in this paper, mathematical knowledge as posited by Plato, consists of axioms and postulates that exist in a separate realm. This problematic view of Plato on the discipline can be related to the question: How is one to know mathematics if numbers exist in a separate realm? Causal connection is an important aspect to be addressed in comprehending Plato’s mathematical forms but this causal relations clearly lacking in his theory. However, an Islamic interpretation of this as espoused by Afridi and Khan (2007) provides a convincing explanation to this problem whereby the causal relationship between numbers, spatial magnitude, mathematics and the soul are made explicit in the following:

*Numberism holds that numbers are things; the development of numbers coincides with the development of things, and the system of numbers with that of the world. One or Unity is the Number. It is the origin, the ground, the father of all members. It is God eternal and immutable. The one, the unity contains in itself the many, the multiplicity; and thus arises time, which is only a plurality of one after the other. The one multiplies itself in the system of numbers. 1+1 is 2, 1+1+1 is 3, 1+1+1+1 is 4. 1 is a point, which is a spatial magnitude; 2 is line, because two points go to make a line; 3 is a surface, because three points construct a figure; 4 is solid, because four points constitute a volume. Thus from number is generated space in all its determinations - The objects in space are solids.*

On the clarifying of how the soul relates to mathematics, Afridi and Khan (2007) have the following to highlight:

*The soul too is number; it is 10; it is harmony of the body. Hence it is capable of knowing (the universe), for the like can know the like; and that clearly by its rationality. Mathematical knowledge alone is knowledge; and knowledge is the highest activity of the soul. In performing this activity the soul is truly happy.........*
Plato’s belief that mathematics exists independently of man and subjected to the divine law of nature. However, this divine element is in itself a rational and an intellectual entity, making God and mathematics belonging to the same realm of the intellect. The connection between logic and metaphysics in the Plato's mathematical forms had never been clearly espoused. The relation between the external world, with its changeable phenomena, and an ultimate, immutable, and unchangeable First Principle still remains obscure.

3. An Islamic Critique of Plato’s Forms

In claiming the forms as having eternal qualities and subsequently tying them with the notion of god, Plato had set forth the belief that mathematical knowledge is symbolic of the ultimate truth of the metaphysical system. This point is made pertinent when Plato asserts that the principles and rules of mathematics depict the precision and perfection of the phenomenal world. The perfection and precision of mathematical knowledge is a valid proof of the relation of a supra-rational entity and its relation with the universe. Thus, perfection and precision of numbers and mathematical forms are indubitable sources of god’s existence as an eternal creator as inferred from the existence of the phenomenal world.

For the purpose of this paper, the writer has no intention of attempting to reconcile Islam with the Greek tradition of Platonic forms as the two do not complement each other. But since the arguments pertaining to god, the soul and forms had been and are still the essential issues addressed by both, this paper will discuss critically on the issues addressed earlier on with emphases on the Islamic interpretation of mathematical principles in relation with the Islamic concept of God. There are some salient points in the Islamic and Greek philosophies that can be discussed, but a point to be clearly noted is, Islamic belief of God is NOT on the same belief dimension as the Greek. Hence, with that in mind, the writers of this paper will attempt to provide an Islamic critique with the intent of setting forth the Islamic justification by first and foremost professing that there is no other God except Allah SWT. As such, the following claim of this paper will be an attempt to highlight the difference between the Greek (Platonic) and the Islamic perspectives pertaining to the ‘forms’.

The first issue to be debated is on the true meaning of the forms themselves which had been subjected to controversies among scholars till the present time. Forms as attested by Plato delve into abstractions of space and number and necessarily needs one to use a language of symbols and figures to represent its concepts. This is consequently followed by logical rules of inference to arrive at the most valid conclusion that is not quite possible in other sciences. However, even though mathematics in the words of Sarwar (1938);

\[\text{is a sort of intellectual jugglery by means of which the Mathematician performs or seems to perform miracles of intellectual feats, he does not and cannot reveal the ultimate nature of the reality of his ideas and concepts.}\]

In relating the above excerpt to Plato’s forms, it can be concluded that the essence of Greek mathematics is description of matters of ultimate reality that are not exhaustively explained. This is so as argued earlier on, Plato attributes at one instance the forms to ideas, at another instance to the soul and also, the forms as unified under the Form of the Good or God. This point is further made confusing when Plato’s master, Socrates (Plato’s work was largely a continuation of the earlier works of his master) claimed the forms to be even more supreme, ultimate and absolute with which the universe is made and governed and not subjugated even to the will of a chief god. For Socrates, the Forms being ultimate, supreme and super-divine are higher than the Gods and remain so eternally. If the human mind can reach the highest level of the intellect and truly understand these forms, then, necessarily, the human mind act in
accordance to the mind of god. Socrates made this point even more pertinent when he posited the claim that the Ideas (forms) are prior to God and existed before him and being supreme, ultimate and absolute, belong to a higher level of reality than God itself. Here is the starting point of controversy of the Greek Hellenistic position that contradicts altogether with the Islamic belief of mathematics, its position in the metaphysical system and the belief in god as the creator of the universe.

Socrates claimed that the universe is not ruled by divine wills, vis-vis divine, eternal Forms that serve both Gods and humans as absolute standards of knowledge and value. Hence, these forms can never be subjected to any divine power or God. Socrates earlier proposition pertaining to the forms contradicts that of Plato, the latter asserting that the lesser forms are unified under the Form of the Good or God. So, putting the views of these two Greek philosophers, there is surely a contradiction between the two beliefs on the forms and the concept of god. Even in Plato’s belief, he attributes at one time forms to ideas and at another time to the soul. Which is his real doctrine has never been clear across the centuries. In fact what is the relation of ‘Ideas’ or ‘Forms’ to Soul had never been convincing as Plato’s approach to these universals was to regard them as real, perfect exemplars or ‘form’ and a priori in nature. ‘Forms’ to Plato can also be supreme moral standard as the forms in Greek mathematics are themselves core properties of the moral forms such as the form of justice. In this context, ‘the Good’ or God by nature prescribes the supreme moral standard and God’s commands reflect perfectly moral nature. Suppose if this claim is true, the controversy that is clearly evident is, how can the forms be more supreme and super-divine than the gods or god themselves/himself? As stated earlier, Socrates maintained that the ideas (Forms) are prior to god and existed before him. Thus, the God necessarily cannot be the most supreme force that creates the universe and the entire celestial system. The ambiguity in Plato’s doctrine of the forms can be inferred in the following remarks by Barrow (pp. 296-297);

The problem of human contact with some spiritual realm, of timelessness, of our inability to capture all with language and symbol – all have their counterparts in the quest for the nature of Platonic mathematics.

On the forms, Nicholas (in Milanese pp. 6) writes, “Knowledge, for Plato, is knowledge of the forms, knowledge has to do with ‘being’ or ‘what is’, namely, Forms.” However, Barrow’s (in Box, 2001) explanation of Platonism mathematics is as follows:

Plato’s philosophy of mathematics grew out of his attempts to understand the relationship between particular things and universal concepts. What we see around the world are particular things – this chair, that big chairs, little chairs, and so on. But the quality they share- let’s call it ‘chairness’ - presents a dilemma. It is not itself a chair and unlike all chairs we know it cannot be located in some place or at some time. But lack of a place in space and time does not mean that ‘chairness’ is an imaginary concept.

The writers of this paper supports to underline the word ‘dilemma’ to highlight the confusing nature of Plato’s forms as shown in the excerpt above. Contrary to the Greek belief, the study of mathematics in the Islamic tradition and belief reveals to us the nature of reality. To quote Sarwar (1938):

“Mathematician performs or seems to perform miracles of intellectual facts/feats BUT does not and cannot reveal the ultimate nature of the reality of his ideas or concepts”.

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Even though mathematics in the Islamic perspective is regarded as the gateway leading from the sensible to the intelligible world, the ladder between the world of change and the heaven of archetypes, with respect to the world of the senses, mathematics is similarly an abstraction (Nasr, 1984). This point which is highlighted in Plato’s world of ideas as a guide to the eternal essences as earlier argued, is confronted with ambiguity and obscurity. The contrast between the two perspectives (Islam and Greek) is the inability of the Platonic doctrine to explain mathematics abstract concepts which are real, self-subsistent and eternal entity in the context of the metaphysical realm with God as the highest form of the Good. Even more confusing is the claim that Demiurge or god is not in any way the creator of the Forms, but, on the contrary dependent on them. On this point, Grube (1992) had convincingly tried to argue,

"by insisting on the necessity of including movement and soul in the ‘real’, Plato is opening the gate by which the gods will enter his dialectical discussions and become an integral part of his philosophic system”. And yet, even the supreme god of the myth is dependent on the Forms.”

The Islamic argument as to the form being subsumed in the metaphysical and mathematical realms can be seen in the following. The concept of the tiniest part of an object had been the subject of debate in the Islamic era of Mutakallimin. They called the tiniest part as al-juz allazi lam yatajaza (the smallest part that could not be further divided). This concept of atom was attributed to philosophical views of both parties in the conflict. The idea of partitioning to the smallest component can be illustrated by the geometric series of

\[ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots \]

Adding all the parts, the whole original object is obtained:

\[ 1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots \]

or

\[ 1 = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \ldots \]

On the debate on atom, one party claimed atom as being the smallest unit of form contrary to the argument of the opposing party that claims that mathematically, an atom is not the smallest form. Ibnu Sina took the stance that atom is not the smallest division since to him, there is still other smaller unit than atom. To rationalise this position taken, he put forward the following proof and geometrical diagram. (Diagram 1).

Diagram 1: Ibnu Sina’s Model to show that atom is not the smallest division or part
In the above diagram, imagine if each circle is an atom and being arranged closely to form an equal measure of a square leaving no space between the atoms. If there is space in between, then atom is not the smallest division or part. Assuming that atom is the smallest unit or part and there is no other unit smaller than it. Based on this assumption, then each of the line and corner hence, consists of 3 atoms. Since there is no space in between each of the atom, the lengths of the boundaries and corners are equal to the length of 3 atoms. This means that AB=BC=CP=DA=CA=DB. The consequence arrived here is definitely impossible as it contradicts with the Pythagoras Theorem. As such, an atom is not the smallest part or division.

The concept of infinity was a matter of great debate during this era as it was related to the concept of metaphysic. In the earlier debate on the concept, Thabit ibnu Qurrah (10th century), had proven that two infinite numbers cannot be compared. Thabit used the method of contradiction to prove his claim, and based on this, an assumption was made that the two infinite numbers can be compared. Geometrically, he illustrated that the first infinite number refers to the line AB and the second infinite number refers to line GD.

Diagram 2: Thabit’s Diagram to negate the comparison of two infinite quantities

A______________F_______________W______________B

G_______________D

A complete proof of this problem can be seen in the following arguments:

Assuming that AB>GD, the two quantities can be compared without reducing its generalization. If AB>GD then GD<AB. This implies that there exists an integer m and p with p<GP, hence,

AB=mGD+p

The conclusion derived shows that geometrically, GD can be eliminated from AB, m number of times with a remaining p. Assuming that AW=mGD, with EW=GD. This means that the quantity of EW is infinite as an addition of p can be made. But, since GD=EW, then GD is also an infinite quantity, which contradicts with the earlier assumption that GD is not an infinite quantity. This contradiction is a series of deduction which assumes that two infinite quantities can be compared. Thus, it is affirmed that two infinite quantities cannot be compared at all.

On the position of mathematics within the cosmic order, Abdul Latiff Samian (1997) also agrees with Plato’s World of Forms as not being dependent upon the human mind and that to Plato, God is the greatest mathematician. Hence, to understand the cosmic order, one needs to grasp mathematical principles. This is made very clear by Abdul Latiff when he asserts in his following remark that Mathematics does not rule out theological principles.

*The relation between the Creator and the physical world is synonymous with the relation of number one and other numbers. Number one is the number from which all other numbers originate. In the same manner, the relation of God and the physical world is patterned. God is the Creator or the One that generates everything in the entire world because God creates everything, from the beginning till the end.*

(pg. 32)
As such, the natural order is a manifestation of God or Allah’s greatness and the mighty display of his generosity and volition. The truths of mathematics thus, are not confined to one’s great mind or the highest intellect but are depictions of Allah’s greatness, Ayatullah as evidenced in the Al-Quran and Sunah. Mathematics, thus is not value-free but embraces concepts of beauty and goodness, precision, accuracy and stability on which the entire world and the universe are created by the sole Creator, Allah SWT. The creation of the universe and the other creations is Allah’s volition or Will in action (Sarwar, 1938). However, forms, colours, quantities, extents are mere outward appearances of the manifestations of reality.

However, the Islamic perspective to an extent, coheres with the Greek-Platonic concept in which Mathematics inquires into abstractions of space and number. This requires one to adopt a language of symbols and figures to represent concepts, following which is the presentation of logical consequences or deductions of the concepts. The whole of mathematics are contained in the axioms, postulates and other a priori truths. As shown in the mathematical calculations above, it can be correctly deduced that infinity is one of the attributes of Allah or God.

All His creation is like the lower terms of the series (A) given above. Each term is part of the Whole or Unity and when God creates, anything infinite He does not in any sense suffer any loss for the simple reason, whether infinitely small or otherwise, all creation (referring to any number of terms in the series (A), are still part of the Whole. Hence, the universe is part of the Will of God. is infinite (Sarwar, 1938, pp.35)

The Platonic - Socratic claim of the Forms of the Good’ or God being the supreme moral standard and reflecting perfectly moral nature as shown earlier in this paper, is likened to the moral conduct of man. However, this is not so in Islam as moral goodness is not synonymous to Goodness of God. Goodness in God or Allah implies beneficence, and blessing which are entirely based on His volition. Man does not possess infinite Goodness which remains the sole attribute and possession of Allah, thus He gives it to whom He pleases and as He pleases. Thus, in the infinite series into which God has spread Himself, each term has its own value (Sarwar, 1938, pp. 43).

If eternity and necessity are essential attributes of God as the Primary Cause, so must they be attributes of His actions, which are His effects. Thus, the conception of God in Muslim philosophy, is definitely not the same as the mathematical forms in the Platonic conception.

Oneness is an attribute of God as He has no partner who participates either in His or in His attributes. This is the Tauhid paradigm which defies the plurality of gods as it contradicts the notion of self-necessity. God is not only the pure Good and whatever goodness he extends to mankind is the fruit of his love and benevolence. So it is very evident that the concept of God in Islam is not the pagan conception of the Greek philosophers. The divine attributes of God according to Ibn Sina are oneness, absolute perfection, pure goodness and God is the contemplator and the contemplated. God has no equal nor opposite. Eternity is therefore an attribute of God only and all realities which are parts of The Real exist in time which is finite (Sarwar, 1938). So are other Quranic attributes of God: omnipotence, omniscience, justice, and generosity.

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