A Conceptual Framework of Acquisition of Mathematical Knowledge to Teach Algebra

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Abstract

This study suggests a conceptual framework of acquisition of mathematical knowledge to teach algebra. This framework highlights the importance of acquisition of knowledge for prospective teachers which generates modes for learning to teach mathematics and introduces a notion of balanced mathematics teaching. A grounded theory approach led to the identification of a matrix of acquisition of prospective teacher’s knowledge for mathematics teaching in terms of: aspects of knowledge; definitions; learning modes, teaching modes, and aspects of students learning were illustrated by examples. Task analysis of Algebra for balanced teaching of prospective teachers at grade-six is also elaborated. Primary purpose of this research was to facilitate learning to teach and to design learning tasks for prospective teachers to transfer knowledge of Algebra so that their pupils assimilate it without any difficulty. Secondary purpose was to investigate how appropriately acquired knowledge of prospective elementary teachers were related with these acquisition modes to develop student’s mathematical thinking. The assessment was done through Mathematics Teacher’s Knowledge Test (MTKT); class room observations and interview checklist. Graphical representations and data analysis showed that educational back ground of prospective teachers is related with the acquired modes. The recommendations have been suggested change in teaching scenario of mathematics teacher education program and for future research.

Introduction

Common wisdom suggests that students' knowledge is influenced by teachers' knowledge. Mathematical knowledge consist of both the knowledge of learning mathematics and the knowledge of teaching mathematics. Mathematical knowledge for teaching is different from the mathematical knowledge used by other specialists. In the process of learning to teach, teachers’ knowledge include both knowledge of mathematics and knowledge of teaching mathematics. Therefore concern of this research is teacher’s knowledge.

Fundamentally concerning with teachers’ knowledge; the first consideration should be the learner of mathematics who might be a pupil in a classroom or a pre-service teacher at university. There exist an expectation that students will learn more mathematics, also how mathematics will be learned in such settings. Secondly how the content knowledge of mathematics grows in the minds of teachers of mathematics. Content knowledge of mathematics is provided by the substantive and syntactic structures of this subject. Generally speaking about the substantive knowledge, it include the knowledge of facts and concepts and the modes in
which it is organized. Syntactic knowledge is about the nature of inquiry in the field and the process through which new knowledge is introduced and accepted in the relevant situation. Schwab(1978) described that syntactic knowledge includes knowledge about proofs and rules of structure.

The implications of knowledge of substantive and syntactic structures for teachers are, how teachers select to teach and also distinguishes between two kinds of understanding: knowing ‘how’ and knowing ‘how to develop student’s mathematical thinking’. Teachers need to acquire enough understanding of the subject to know which ideas are central, which are peripheral, how different ideas relate to one another and how these ideas can be represented for students appropriate learning.

Algebra is the branch of mathematics that deals with numbers and their relations. In school mathematics algebra is used to generalize results in arithmetic. As it has been called a generalized arithmetic, so it may be related to geometry saying that algebra is only written geometry and geometry is merely pictured algebra. In teaching and learning, its principles are frequently needed to corresponding situations in arithmetic. Algebra is used in people’s daily lives from buying groceries in the store to scientific researches. It gives new approaches to the study of abstract mathematical relationships through the use of a new language and a new symbolism. Verification of results is simpler and more satisfactory in algebra than in any other branch of mathematics. It develops confidence among the students and a good instrument for mental training.

Algebra is considered to be a challenge for many people who don’t understand the language of mathematics and a gatekeeper for those who want to learn mathematics. Successful understanding of algebra course is not only considered a prerequisite for further study in mathematics and in other subjects also but open the door for many opportunities of jobs. Algebra is a tool that provide the power to the learner for operating and understanding the concepts at abstract levels and then their applications in other scientific disciplines. Algebra can never be considered only as a branch of mathematics, it plays an important role for the mathematics proficiency. The students experienced many difficulties with algebra when they feel a big change while operating with symbol which represent particular numbers in arithmetic. The fact that a large portion of students do not acquire the expected levels of proficiency, even after several years of study, is of grave concern to educators. Therefore the difficulties encountered in teaching and learning of algebra also delays the progress in the learning of higher level mathematics. So as far as personal teaching experiences are concerned and researches conducted on algebra learning and teaching have shown that the gateway to algebra is never as smooth as one may wish. Bednarz, Kieran, C. & Lee, L.(1996) stated that during learning to teach algebraic tasks are little bit difficult. There are several complexities that might make algebra difficult to understand: the content of algebra, the modes of teaching algebra, inappropriate approaches for developing on or students’ mathematical thinking. The teacher of mathematics might have a central role to prevent, lessen, or deal with the difficulties student encounter. This research is designed to address acquisition of mathematical knowledge and understanding of teacher perception of algebra (within mathematics and schooling), teaching beliefs and developing students’ understanding in algebra.

Although elementary teachers are a critical part of algebra reform they typically have little experience with the rich and connected kinds of algebra that need to become the norm in their
instruction. The adequacy of elementary teachers’ substantive and syntactic knowledge of mathematics, for their own professional purposes, cannot by any means be taken for granted.

Most school mathematics curricula are designed with developing computational algorithms rather than the development of higher order thought processes, such as those used in problem solving, deductive reasoning and logical inference. This is true throughout the elementary, junior high and senior high school levels. For example, at the elementary level, students are found practicing the long division algorithm, while at higher levels they spend considerable time on the manipulation of algebraic sentences or on solving quadratic equations. For the development of higher order thought process, it is needed that student learn more algebra, which is only possible when elementary teachers are fully equipped with basic knowledge to transfer knowledge of algebra so that their pupils assimilate it without any difficulty.
Significance of Research

The advancement of modern technology has paved the way towards new approaches. The current mathematics education reform efforts for teacher preparation, emphasize the adequacy of teachers’ mathematical knowledge and the development of content and pedagogical practices, the questions strike the mind that how teachers acquire mathematical knowledge for the adequate development of pedagogical practices. Ebel (1982) described that how to acquire knowledge and learning how to think could seem to be similar goals. It is simple that one can’t assimilate knowledge without knowing how to think and assimilated knowledge is deserving to be treated in the same way. A mind full with the rote memorization of facts possesses very little useful knowledge.

Objectives of Research

The objectives of research were as follow:

- To facilitate learning to teach mathematics.
- To make efforts in the specific cognition of mathematical topics.
- To provide opportunities for the prospective (pre-service) teachers to design learning task.
- To enable the student teachers to transfer subject matter knowledge of algebra so that their pupils would assimilate it without any difficulty.

The secondary purpose was to apply it on the acquisition of pre-service teacher’s knowledge for teaching Mathematics focusing on student’s learning in basic algebra of sixth-grade.

Research Questions

This research had investigated the following questions.

Q1. Does the acquisition of knowledge is important to mathematics for all prospective teachers?

Q2. What are the various aspects of acquiring the mathematical knowledge for teaching mathematics?

Q3. How appropriately acquired knowledge is used by the teachers to understand and develop students mathematical thinking?

Knowledge Acquisition Theories

A major issue in the theory of cognitive development has been whether it occurs in stages or is continuous. Piaget (1952) and other neo-Piagetian theorists have argued that cognitive development passed through a series of discrete stages. Although Vygotsky (1962) was not a stage theorist, he argued for zone of proximal development, which corresponds to the improvement that instruction can produce on the child’s current knowledge stage. It implies that new knowledge can only extend a limited distance beyond existing knowledge. Other theorist argued that cognitive development depended on acquisition and reorganization of knowledge. There are many theories of knowledge acquisition regarding cognition in which theorists have
attempted to give the imagination, how the learner acquires knowledge and how it is interpreted in his mind. The more critically accepted theories are; schema theory and cognitive flexibility theory.

Schema Theory:- To understand the cognitive process, theoretical framework was designed in the form of Schema theory. This theory suggests that when the learners obtained knowledge, they try to fit that knowledge and structure it in their memory which helps and enables to make sense of memorized knowledge. According to this theory learner break down the obtained information into generalized chunks which are completely and directly stored in the brain and latter on can recall them. That’s why it is an active strategy coding technique which is necessary for facilitating the recall of knowledge. According to Gagne (1985) when new knowledge is obtained, it is expressed in different and indirect way into either already existing schema or organized into a new script written in the mind. In general, schemata are organized pieces of information that support the learner’s ability, how to understand and associate what is being offered to them. The beliefs of schema theory was the first introduced by Piaget (1960) and generated the concepts of accommodation and assimilation. Today, these concepts are described as assimilation being schema usage and accommodation as schema change. Ausubel, D. (1963) extended these beliefs to refine the elements which include gradual production of accumulation of factual information within the an existing schema. Many researchers argued that on the structuring of knowledge and its conceptualization can be seen globally or it may be also for the specific domain. Piaget’s theory of child development is the best example of Global structuring, which define the child’s four stages of intellectual development (sensorimotor, preoperational, concrete operational and formal operations), he also suggested that the process of changing between each occurs because of a fundamentally restructuring of information. Due to which children are able to acquire knowledge in all situations. Although most of researchers argue that when a child becomes an expert of the knowledge then he possesses the ability of acquiring knowledge in all domains. Beginners in that field have different views about it and the modification and adjustment of knowledge in different domains cannot be done by them. During last ten years, Halford (1992) discussed that the capability to make knowledge practical and useful is associated with how the knowledge is first received.

Cognitive Flexibility Theory:- Cognitive Flexibility Theory (CFT) is also like schema theory of knowledge acquisition and its transferring. Dienese (1964) suggested that general schema theories might be useful work for introductory level. Wearne (1986) state that misconceptions might be happened due to the oversimplified perspective of the topics . The advancement in the knowledge acquisition, interconnections exist among the concepts. Therefore flexibility is needed in learning of complex ideas. Also multiple representations and visual learning can develop the flexibility in learning the complex knowledge.

Multiple representations: Through the usage of multiple representations best knowledge is acquired by the learner in the complex domain of knowledge acquisition, the individual should be able to learn the knowledge from multiple perspectives. This would be the ability of mind of learner to make it flexible and may easily apply it into different situations. Furthermore, cognitive flexibility theory is based upon studies that describe the effect of oversimplification of knowledge on the students ability to learn advanced knowledge and also state that over simplification promotes misconceptions of concepts among students.

Visual Learning: Miller, (1983) stressed on visual learning. Visual learning possess two stages: the first stage of visual learning is differentiation in which categorization and classification
information are to be happen, interpretation is the second stage, it involves synthesizing of knowledge for making conclusions and judgments about the newly integrated information. Besides these, for interpreting the information creativity plays an important role. Understanding and the meanings of visual can be enhanced due creativity. In Cognitive development, for knowledge acquisition purposes the usage of visuals does not come without concerning the prior experiences and expectations mostly effect it.

Mathematics educators generally agree that knowledge of concepts is the foundation for intuitions and procedures and that a teacher should be concerned with the development of both conceptual and procedural knowledge. Lawler (1981) discussed that acquisition of mathematical knowledge require deep understanding of procedural and conceptual knowledge and relationship among them. Relationships can be established on two levels. On one level, understanding originates from the ideas presented within the context which is called context-specific understanding. On the second level, relationships are understood in an environment where appropriate abstractions have been made, that is connection-specific.

Algebra is the language of mathematics. It opens doors to more advanced mathematical topics for those who master basic algebraic concepts. It closes doors to college and to technology-based careers for those who do not. Those most seriously affected by lack of algebraic skills are students from minority groups.

Kutz (1991) in recent reform efforts across the nation have addressed the need for all students to learn algebra. Algebra is typically considered the “gatekeeper” for higher math courses, college, and many vocational careers. Prospective elementary teachers when enter in training courses with some technical skill in algebra. They cannot appropriately manipulate familiar types of polynomial, rational, and exponential expressions to solve equations and inequalities and to transform given expressions into equivalent forms. They also have rudimentary knowledge of number theory concepts like primes, factors, multiples, greatest common divisors, and least common multiples, and they are not proficient in operations with fractions, decimals. However, prospective teachers tend to have only limited understanding of the algebraic properties. As awareness of the crisis in mathematics education has grown, “Algebra for All” has become a widely supported goal that is endorsed by the National Council of Teachers of Mathematics(2000) that, “No Child Left Behind Act(2002)”. Consequently, implementing higher math standards which including the requirement that all students complete an algebra course prior to graduation. As a result, those students who are struggling with basic mathematical concepts and skills face new challenges. How schools and teachers address these challenges determines the extent to which these students have the opportunity to succeed in mathematics, graduate from high school, and successfully move forward into meaningful careers or higher education (Ma, 1999). The evidence that students are not learning algebra has sparked new research into the causes of algebra difficulties and effective practices in algebra instruction. Much of this research recognizes that difficulty in algebra stems from students’ inadequate mastery of the underlying mathematics concepts and skills—mastery that is critical to the subsequent attainment of algebra proficiency. Researchers also indicated that effective instructional methods for algebra include direct instruction, guided learning and practice, appropriate sequencing of problems, strategy instruction in step-by-step procedures for problem-solving, and student assessment and observation. These components of instruction are absent in many widely used curricula.

A challenge facing the field is how best to design a framework for acquisition mode to help teachers and better their understanding of the underlying mathematics concepts, and skills as
well as their knowledge of teaching mathematics especially balanced practices in algebra instruction.

**Definition of acquisition of knowledge**

The process of learning to teach mathematics is two dimensional in which prospective teachers learn mathematical knowledge i.e., knowledge of mathematics and teaching mathematics. Therefore there are two dimensions of definition of acquisition of knowledge in the context of learning to teach mathematics:

- In learning context the act of getting knowledge adds to what the teachers already know about mathematics.
- In teaching context ability or behavior that teachers develop by their own efforts during their course of professional training.

Learning to teach is a process of bringing about changes in the individual in desired directions, for the development of interest, attitude and skills in the students. In the professional training of teachers these changes are carried out by learn to do certain activities. This helps the teachers to lead happy, productive and socially acceptable life. Also learn to design activity provides the purposefulness of educational process.

Considering both the dimensions of learning and teaching of mathematical knowledge of preservice teachers, particularly in professional development of prospective teachers for elementary algebra teaching. The prospective teachers who enroll for teacher training courses have normally inadequate teaching experiences and limited mathematical understandings. The mathematical knowledge they possess at entry level of their courses have three aspects: substantive knowledge of mathematics; knowledge of nature and practices of mathematics and the knowledge of connections. During these training programs prospective teachers are considered to develop an understanding of these aspects of knowledge, so that to become scientific bent of mind. This does not mean that teachers should have only, the knowledge of mathematics but he should be systematic and orderly acquiring to do it. When the teachers do it with understanding they would become practical mind.

As learning mathematics is a rigorous process and individual learn throughout his life, therefore a teacher can never teach truly unless he is continuously learning himself. Any teacher of mathematics can continue to grow and be effective in his or her work, if he is aware of what is new and changing in the field of mathematics. Therefore it is important for prospective teachers to recognize the different approaches of acquiring mathematical knowledge called the acquisition modes. The word “mode” is derived from the methods or way. In learning to teach mathematics the acquisition modes play important role for imparting mathematical knowledge to prospective teachers. Though learning to teach is an art. Therefore acquisition modes are the way to understand and practice this art.

**Construction and administration of Instruments**

Three instruments were constructed as: Questionnaire named Mathematics teachers’ knowledge test (MTKT); Checklist for classroom observations; and Checklist for interviews.

**Construction of questionnaire named Mathematics teachers’ knowledge test (MTKT)**

In constructing the items, techniques of construction and the use of questionnaires were studied. Rimmer while discussing about opinion and attitude measurement, emphasized the importance of
questionnaire and checklists in education research. He said that the educators, especially those involved in guiding the educational activities, were making more and more use of these devices.

For assessing the acquisition of prospective teachers’ knowledge MTKT was designed according to the pattern of Test of Teaching knowledge (TTK) of Cambridge University. MTKT was focused on three text book problems of Algebra topics (Algebraic Expression, Co-efficient, base and Exponent) of grade-six. The items of these three problems were constructed to assess the acquisition modes of each aspects of knowledge respectively. These were described as below:

**Problem 1**: Assessing the acquisition modes of teachers’ knowledge for building on students’ math ideas.

**Problem 2**: Assessing the acquisition modes of teachers’ knowledge for engaging students in math learning.

**Problem 3**: Assessing the acquisition modes of teachers’ knowledge for promoting students thinking about mathematics.

Each problem consisted of three questions of algebra topics having four choices for each question as shown in see Appendix. A. The assessment of acquired modes of teachers’ knowledge was based on: awareness of the Mathematics curricula of grade-six of Punjab Text book board Lahore; algebraic topics, i.e. Concept of constant, Variables, Algebraic expression, Co-efficient, Base and Power.

**Administration of MTKT**

Hundred copies of questionnaire were administered to the prospective elementary teachers of the three campuses of University of Education. The list of names of participants are given in Appendix D. In order to achieve reliable responses, ability and willingness of the respondents were sought. Besides this, other characteristics of the participants were: it included those who were studying in the B.Ed (General) course; Qualified at least 1st semester of B. Ed program; mathematics is imperative in this program; all had a bachelor degree; and had designed fifteen lessons for teaching, at least five lessons on topics of algebra of sixth-grade.

**Construction of Checklists of Classroom Observations and interviews**

To support MTKT, two checklists, one for classroom observations and the other for interviews were constructed with a two point scale for acquisition modes: appropriate, inappropriate. After the administration of MTKT and collecting the data from the prospective teachers, items of MTKT were classified. This classification was grouped according to the acquired modes of teachers’ knowledge. Classification was then categorized for the checklists of classroom observations and interviews. Categories was made to confirm that teaching practice match with their responses on MTKT.

**Administration of Checklists**

Category of response were observed during teaching practice of algebra topics with two point scale: appropriate/inappropriate acquisition mode. After observing the teaching practices of each participant, an interview was conducted and noted by marking tick on appropriate/inappropriate acquisition mode.
DISCUSSION

In this rapidly changing world, younger nations like the older ones, face challenges which call for the greater knowledge and greater willingness to learn new ways.

Most of the educationists consider that mathematics is an important subject and it holds an important place in primary and secondary schools. Mathematics develops the child as social and intellectual citizens, like other subject discipline. It has its own disciplinary values. In addition to these, mathematics also develops those qualities which can be developed by other subjects. Napoleon also remarked that, “The progress and improvement of Mathematics is linked it the prosperity of the state”. Therefore it is important to have knowledge of mathematics, it is surely also important that students should have its deeper understanding.

It is a common view that students’ knowledge is influenced by teachers’ knowledge. Teachers are responsible to make teaching-learning process interesting and understandable for students being important to impart knowledge. Therefore it is necessary to enhance the acquisition of teachers’ mathematical knowledge.

The purpose of this study was to facilitate learning to teach mathematics, and the design of study was delimited to pre-service teachers for learning task Algebra so that the students could assimilate it without any difficulty. Modes of acquiring mathematical knowledge were sorted out. A conceptual framework of acquisition of knowledge for pre-service teachers, based on objectives of learning and teaching mathematics was chalked out, modes of acquiring mathematical knowledge were gleaned from related literature.

A critical review of literature with the purpose to determine the modes of acquiring teachers’ mathematical knowledge was made in various categories as: theories of the knowledge acquisition and its acquisition for students learning; theories and practices in teaching-learning context was; acquisition of mathematical knowledge, knowledge of teaching mathematics; knowledge of Algebra; and teaching of Algebra. For designing the conceptual framework and for the assessment of acquisition of teachers’ mathematical knowledge, besides this the techniques of construction and validation of questionnaire, checklist of classroom observations and interviews checklist were also studied.

The review of literature emphasize the theories of knowledge acquisition conceptualizing how knowledge is acquired and interpreted in the mind of learner; while in psychological perspective knowledge acquisition is the ability to acquire mathematical knowledge and how it is internally represented. But acquisition of knowledge in learning to teach, is the connection of prospective teachers’ prior knowledge to new knowledge. Therefore three essential aspects of teachers’ mathematical knowledge are, substantive knowledge of mathematics, knowledge of nature and practice of mathematics and the knowledge of connections in learning and teaching context.

The definition of acquisition of mathematical knowledge has been given in conceptual framework. During learning to teach process teachers acquire two types of acquisition modes of mathematical knowledge: learning mathematical knowledge with understanding and the other is acquiring teaching strategies to develop students’ mathematical thinking. Learning modes entails prospective teachers to learn mathematical knowledge with understanding, while teaching modes enables them to acquire the teaching strategies to developing students’ mathematical thinking. This has been shown that learning modes require a great deal of understanding mathematical knowledge. This has been shown in it that learning modes require a great deal of understanding mathematical knowledge. These conceptual basis for learning modes has been also represented in concentric circles with center at understanding mathematical knowledge. A model activity of painted cube problem was discussed to show that how teachers learn to apply conceptual basis.
for developing modes of learning Algebra. The relationship of both acquisition modes were represented as a net work of acquisition of teachers’ mathematical knowledge for developing students’ mathematical thinking has been expressed in a bucket according to Chinese saying: “If you (the teachers) want to give the students one cup of water you (the teachers) should have a bucket of your own.” A matrix of acquisition of teachers’ mathematical knowledge has been designed which has been consisting of: Aspects of mathematical knowledge; Definitions, Learning modes; Teaching modes; aspects of students’ learning with examples. Considerable importance has been given to pre-service elementary teachers since elementary teachers develop the foundation of students in mathematics. The aspects of knowledge of Algebra of grade-six have been given importance, because elementary Algebra is the base for higher Algebra courses in the secondary and higher secondary schools. Therefore more attention has been paid to elementary teachers’ knowledge of Algebra.

To assess the elementary teachers’ modes of acquisition, an instrument mathematics teachers’ knowledge test questionnaire (MTKT) was designed for the knowledge of Algebra. Ten multiple choice questions each having four choices on three text book problems of Algebra of grade-six were constructed to assess prospective elementary teachers acquired modes. To support MTKT two checklists, one for classroom observations and other for interview were also designed against a two point scale for acquisition modes: appropriate, inappropriate. The responses on items of MTKT were computed. The responses to checklists of classroom observations and interviews were also tabulated. To find out the nature of responses on the items of MTKT, the number and the 100 % responses on each items were also computed. The classification of teachers’ qualification were made in nine groups and data were classified and from table 1 to table 5 and tabulated from table 6 to table14. The relationship between acquired modes and teachers’ knowledge (qualification) by representations of vertical bar graphs had been drawn and to support this, a relationship between classroom observations and interviews was found by Pearson formula.

Results

Results of responses to mathematics teachers’ knowledge test (MTKT) was tabulated and graphically represented in nine groups. The percentage of responses of nine groups of prospective teachers of mathematics teacher’s knowledge test (MTKT) are presented in vertical bar graphs. Teacher’s responses were coded in percentage for each category. On the basis of the data obtained from the analysis of responses and within the limitations of this study, the following results were drawn:

1) For the appropriate acquisition of mathematical knowledge, the prospective elementary teachers should have mathematics as a part of their academic background.

2) Responses have shown graphically that prospective teachers having sound mathematical back ground acquired 100 % modes for focusing on students’ learning.

3) High correlation between classroom observations and interviews indicated that prospective elementary teachers’ teaching practice is consistent with the acquisition modes, teachers acquire during their one year training.

4) It was found that understanding mathematics is very important along with the methodology of teaching.

5) The hierarchy of tasks can produce learning with understanding of topics to be learned and taught.

6) Elementary teachers should have to practice the conceptual basis for the appropriate acquisition of knowledge, during their training.
7) Teachers should have to acquire strategies of teaching for developing students’ mathematical thinking.
8) Prospective teachers should have to follow the current trends in teaching of mathematics.
9) Appropriately acquired mathematical knowledge enhances the professional growth.
10) Prospective teachers should be trained in the supportive learning environment.
11) Supportive learning environment in learning to teach mathematics produces teaching style more expanded and creative.
12) Role of a teacher should be more of a facilitator of learning than importer of knowledge.

Discussions

The arenas for testing the given ideas are in teacher education program and teaching of mathematics. In teacher education, if there is appropriate acquisition of mathematical knowledge, as this study suggests, then it would be possible for programs to facilitate teachers to acquire modes of learning and teaching mathematics. To probe this, it was investigated that whether prospective elementary teachers have learned mathematical knowledge with understanding or in a relatively traditional way. During the classroom observations and interviews with these prospective teachers of mathematics. It was found that teachers did acquire both modes of acquisition of knowledge as a result of their mathematical background. Also it was found that 100% responses on items of MTKT were related to their algebra knowledge of curricula of grade-six. In addition to these specific findings, this study sets the stages for future analysis of the environment under which teachers acquire appropriate mathematical knowledge.

One of the most pressing issue is whether elementary teachers can acquire modes of acquisition without having mathematical background. The analysis of data from responses on items of MTKT, classroom observations and interviews with the teachers showed that answer may be yes. It was found that during teaching practice, teachers tried to interact with students by performing manipulative activities, pose questions, give examples from daily life, use rules & procedures and develop a supportive learning environment which made their teaching style more expanded and creative. This suggests that there should be a place in the professional preparation for involving prospective teachers in manipulative activities to enhance their mathematical understanding. For the enhancement of teachers’ understanding of mathematical knowledge the appropriate acquisition of knowledge is essential. Therefore the acquisition of knowledge bases for the prospective teachers as suggested by Schulman, can be enhanced, if the teachers’ acquire modes of learning to teach mathematics.

In teaching of mathematics the main question was, how appropriately acquired modes are related with the teachers’ mathematical knowledge, and do the teachers acquire 100% acquisition modes for developing students mathematical thinking? To answer these questions, MTKT was designed in such a way that allowed to test assumptions empirically. Based on study of practice as well as the research on learning and teaching mathematics, analysis of curriculum materials, examples for students learning mathematics and personal experiences, forty multiple choices were designed to assess acquisition modes for learning and teaching mathematics. Building a good item from early idea stage to finish (Reviewed, revised, pilot tested and analysis) takes a lot of time. In spite of this assessments, this research also identifies modes of acquisition of mathematical knowledge and also how to develop students’ mathematical thinking. These modes can be applied in any other subject discipline of science education and equally applicable in other mathematically intensive profession such as accounting or engineering. This claims that appropriate acquisition of mathematical knowledge is related to high quality mathematics teaching.
Most important purpose of this research was, do the acquired modes are correlated to the educational back ground of prospective teachers? Is there any appropriate acquisition of knowledge? What does our study suggest? The answer to these questions are the graphical analysis of percentage of responses on the items of MTKT. The graphical representation of acquired modes and percentage of responses were shown in chapter 6. The bar graph of group (iv) & (v) shows that mathematically literate teachers acquired 100% modes of acquisition. This strengthens the assumption empirically that appropriately acquired modes are highly related with the teachers’ mathematical knowledge and these teachers has shown 100% responses for developing students’ mathematical thinking.

More work is required in this area. Does acquisition of teachers’ mathematical knowledge as conceptualized, positively gain in students learning. Do the acquisition of teachers’ knowledge affect the students’ math learning? Can the enhancement of teachers’ knowledge be the factor of achievement of students’ learning?

Conclusions

This research has yielded substantiate answers to the questions that derived from the current discussion about acquisition of teachers’ mathematical knowledge for teaching Algebra at elementary level. Also on the bases of the results from the analysis of especially designed MTKT, classroom observations and interviews within the limits of this study, it is evident that prospective teachers appropriate acquisition can produce educational balance, effectiveness and enhancement of the subject discipline. So the following conclusions were drawn:

- A balance in learning and teaching modes is essential for understanding and achieving mathematical proficiencies.
- The major goal of teacher education program should be to produce critical thinkers and the active independent learners rather than static learners.
- Teaching is more than dispensing information because learning is more than receiving.
- It is also possible to develop and validate a framework that will establish guidelines for the acquisition of teachers’ mathematical knowledge.
- The acquired knowledge of prospective elementary teachers can be assessed by designing the test items, classroom observations and interviews.
- Prospective elementary teachers mathematical knowledge is highly related with the appropriate acquisition of mathematical knowledge they acquire during their training.
- Content and process can be taught simultaneously, what is learned is tied to how one learns.

Recommendations

Our society needs the teacher education program to produce such trained prospective who will not only fulfill the manpower requirements but also acquire necessary insight and skill for the future shocks. To keep pace with the social, economic and industrial change the teacher must acquire up dated knowledge of the discipline to become a good communicator and efficient organizer. Hence in the changing scenario the role of teacher must be a more facilitator for students learning.

In the light of changing views about how children learn mathematics and how the modes of teaching mathematics facilitate students learning, During training prospective teacher should learn to design activities for the enhancement and development of mathematical ideas on students. In essence, the emphasis must be placed on how to develop students’ mathematical
thinking. Keeping these in mind, six distinct trends in the teaching of mathematics were recommended in the figure 12 as:-

1. Information is presented in such a way that children can use prior knowledge to reinforce and expand previous understandings and construct new ones.
2. Children are given varied and numerous opportunities to act on their environment both mentally and physically in order to define and solve problems.
3. Adults help children to define problems, seek answers, and organize information.
4. Children are encouraged to share ideas and interact with another.
5. Information presented in the classroom is relevant to children’s lives, so that they can apply it to real–world situations.
6. Children are given experiences that help them relate concepts in one content area to other concepts in the same area and to concepts in other content areas.

Figure: 12   Modes of Teaching Mathematics for Changing Scenario.

Secondly, in the light of results of the study, following recommendations were suggested for future in teacher education program:-

- Set up dimensions for the cross cultural comparative studies, although these modes of acquisition of knowledge are the parts of learning to teach mathematics in our cultural context.
- More researches should be conducted to explore the ideas of acquisition modes in any other science discipline other than mathematics.
- More emphasis should be given on the acquisition of knowledge during the teachers training program rather than pedagogical skill.
References


